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| System of interest |  |  |
|  |
| Without noise | | |
| SINDy-PI | Synthetic data |  |
| SINDy-PI Model |  |
| x0\_dot = 0.372 x1 + 0.034 x0x1 + 0.001 x0x0 + 0.017 x1x1 + 0.052 x1x1x1 + 0.001 x0x1x1x1 + 0.066 x0x0\_dot + 0.060 x1x0\_dot + 0.040 x0x0x0\_dot + 0.062 x1x1x0\_dot + -0.001 x0x0x0x0x0x0\_dot + 0.204 x1x1\_dot + 0.019 x1x1x1\_dot + 0.004 x1x1x1x1\_dot + 0.005 x0x1x1x1x1\_dot  x1\_dot = 0.007 x0x0x0x0 + -0.003 x0x0x0x0x0 + 0.011 x0x1x1x1x0\_dot + -0.013 x0x0x0x0x0\_dot + 0.005 x0x0x0x0x0x0\_dot + -0.002 x1x1x1x1x1x1x0\_dot + 1.509 x1x1\_dot + -0.291 x0x1x1\_dot + -1.149 x1x1x1\_dot + 0.250 x0x1x1x1x1\_dot + 0.043 x0x0x0x0x1\_dot + -0.014 x0x0x0x0x0x1\_dot + 0.001 x0x0x0x0x0x0x1\_dot + 0.040 x1x1x1x1x1x1x1\_dot + 0.689 sin(x0x1)x1\_dot |

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| Without noise | | |
| Ensemble-SINDy | Synthetic data |  |
| Ensemble-SINDy Model |  |
| (x0)' = 1.000 x1  (x1)' = 3.016 1 + -0.606 x0 + -0.996 x1^2 |

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| With noise | | |
| SINDy-PI | Synthetic data - original |  |
| Synthetic data with noise  1% rmse |  |
| SINDy-PI model |  |
| x0\_dot = 9.248 x0 + -1.620 x1 + 0.286 x0x1 + -13.736 x0x0 + 0.868 x1x1 + -0.189 x0x1x1 + 7.169 x0x0x0 + -0.045 x1x1x1 + 0.012 x0x1x1x1 + -1.708 x0x0x0x0 + 0.095 x1x1x1x1 + 0.190 x0x0x0x0x0 + 0.029 x1x1x1x1x1 + -0.008 x0x0x0x0x0x0 + -0.023 x1x1x1x1x1x1 + -0.314 x1\_dot + 0.388 x0x0\_dot + 0.299 x1x0\_dot + -0.055 x0x1x0\_dot + 0.323 x0x0x0\_dot + 0.206 x1x1x0\_dot + -0.034 x0x1x1x0\_dot + -0.270 x0x0x0x0\_dot + -0.092 x1x1x1x0\_dot + 0.014 x0x1x1x1x0\_dot + 0.075 x0x0x0x0x0\_dot + -0.017 x1x1x1x1x0\_dot + -0.009 x0x0x0x0x0x0\_dot + 0.003 x1x1x1x1x1x0\_dot + 0.001 x1x1x1x1x1x1x0\_dot + 0.093 x0x1\_dot + 0.174 x1x1\_dot + -0.031 x0x1x1\_dot + 0.095 x0x0x1\_dot + 0.034 x1x1x1\_dot + -0.006 x0x1x1x1\_dot + -0.066 x0x0x0x1\_dot + -0.045 x1x1x1x1\_dot + 0.005 x0x1x1x1x1\_dot + 0.017 x0x0x0x0x1\_dot + -0.001 x1x1x1x1x1\_dot + -0.002 x0x0x0x0x0x1\_dot + 0.003 x1x1x1x1x1x1\_dot  x1\_dot = 8.696 x0 + 0.697 x1 + -0.168 x0x1 + -12.411 x0x0 + -0.840 x1x1 + 0.033 x0x1x1 + 6.544 x0x0x0 + -0.235 x1x1x1 + 0.037 x0x1x1x1 + -1.597 x0x0x0x0 + 0.236 x1x1x1x1 + 0.182 x0x0x0x0x0 + 0.020 x1x1x1x1x1 + -0.008 x0x0x0x0x0x0 + -0.018 x1x1x1x1x1x1 + -0.294 x0\_dot + 0.042 x0x0\_dot + 0.150 x1x0\_dot + -0.027 x0x1x0\_dot + 0.196 x0x0x0\_dot + 0.022 x1x1x0\_dot + -0.003 x0x1x1x0\_dot + -0.129 x0x0x0x0\_dot + -0.033 x1x1x1x0\_dot + 0.004 x0x1x1x1x0\_dot + 0.033 x0x0x0x0x0\_dot + -0.003 x1x1x1x1x0\_dot + -0.004 x0x0x0x0x0x0\_dot + 0.002 x1x1x1x1x1x0\_dot + 0.457 x0x1\_dot + 0.283 x1x1\_dot + -0.054 x0x1x1\_dot + 0.182 x0x0x1\_dot + 0.211 x1x1x1\_dot + -0.035 x0x1x1x1\_dot + -0.184 x0x0x0x1\_dot + -0.080 x1x1x1x1\_dot + 0.014 x0x1x1x1x1\_dot + 0.052 x0x0x0x0x1\_dot + -0.015 x1x1x1x1x1\_dot + -0.006 x0x0x0x0x0x1\_dot + 0.001 x1x1x1x1x1x1\_dot + 0.001 x1x1x1x1x1x1x1\_dot |

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| Ensemble-SINDy | Synthetic data original |  |
| Synthetic data with noise  0.1% rmse |  |
| Ensemble-SINDy model |  |
| (x0)' = 1.000 x1  (x1)' = 3.111 1 + -0.622 x0 + -0.374 x1 + -0.753 x1^2 |
| Ensemble-SINDy | Synthetic data original |  |
| Synthetic data with noise  1% rmse |  |
| Ensemble-SINDy model |  |
| (x0)' = 0.855 1 + -0.170 x0 + -0.133 x1 + 0.181 x0 x1 + 0.291 x1^2  (x1)' = 2.634 1 + -0.525 x0 + 0.365 x1 + -0.133 x0 x1 + -0.951 x1^2 |
| Synthetic data with noise  10% rmse |  |
| Ensemble-sindy model |  |
| (x0)' = 5.302 1 + -1.788 x0 + -9.797 x1 + 0.133 x0^2 + 1.574 x0 x1 + 4.103 x1^2  (x1)' = 3.428 x1 + -0.621 x0 x1 + -1.546 x1^2 |

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| Ensemble-SINDy | Synthetic data original |  | |
| Synthetic data with noise  0.5% gaussian w/ mean = 0, |  | |
| Ensemble-SINDy model |  | |
| Ensemble with replacement (V1)  (x0)' = 1.090 x1  (x1)' = 2.968 1 + -0.608 x0 + -0.551 x1 + -0.529 x1^2 | Ensemble model without replacement (V2)  (x0)' = 0.998 x1  (x1)' = 3.102 1 + -0.620 x0 + -0.385 x1 + -0.741 x1^2 |

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| Ensemble-SINDy | Synthetic data original |  | |
| Synthetic data with noise  Gaussian w/ mean = 0, and std dev = 0.01 |  | |
| Ensemble-SINDy model |  | |
| Ensemble with replacement (V1)  (x0)' = -0.560 1 + 0.792 x0 + 1.997 x1 + -0.135 x0^2 + -0.264 x0 x1 + -0.480 x1^2  (x1)' = 2.392 1 + -0.481 x0 + 0.977 x1 + -0.207 x0 x1 + -1.165 x1^2 | Ensemble model without replacement (V2)  (x0)' = 0.999 x1  (x1)' = 2.526 1 + -0.504 x0 + 0.528 x1 + -0.160 x0 x1 + -0.999 x1^2 |

**Adding less noise**

20/07: followed a new example for using SINDy-PI (github changed so the previously used code for SINDy-PI isn’t working)

Filename: spring\_damp\_sin\_sindypi.py

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| **Test** | **Output** | **Comment** |
| Solving the spring mass damper system with a sin term in SINDy-PI – no noise | 1 = 0.3782 x0 + -0.0357 x0x0 + 0.3513 x0\_t + 0.2517 x0\_tt + 0.0031 x0x0\_t + -0.0146 x0x0x0\_t + -0.0092 x0x0x0\_tt  x0 = 2.0059 1 + 0.1204 x0x0 + -0.4262 x0\_t + -0.0369 x0\_tt + 0.0187 x0x0x0\_t + -0.5094 x0x0\_tt + 0.1105 x0x0x0\_tt  x0x0 = -22.1064 1 + 9.4118 x0 + 9.8645 x0\_t + 5.2465 x0\_tt + -2.1817 x0x0\_t + 0.0360 x0x0x0\_t + -0.2309 x0x0x0\_tt  x0\_t = -0.1238 x0 + 0.0246 x0x0 + 0.7650 x0x0\_t + -0.1129 x0x0x0\_t + 0.1777 x0x0\_tt + -0.0304 x0x0x0\_tt  x0\_tt = 0.4921 1 + -0.0204 x0x0 + -0.3520 x0\_t + 0.0136 x0x0x0\_t + 0.8995 x0x0\_tt + -0.1473 x0x0x0\_tt  x0x0\_t = 0.3021 x0 + -0.0603 x0x0 + 1.0716 x0\_t + 0.1559 x0x0x0\_t + -0.2055 x0x0\_tt + 0.0293 x0x0x0\_tt  x0x0x0\_t = -3.9906 1 + 0.1572 x0x0 + -5.4808 x0\_t + -0.4449 x0\_tt + 6.1278 x0x0\_t + 2.8344 x0x0\_tt + -0.4865 x0x0x0\_tt  x0x0\_tt = -0.1940 x0 + 0.0396 x0x0 + 0.2663 x0\_t + 0.9231 x0\_tt + -0.0518 x0x0\_t + 0.1705 x0x0x0\_tt  x0x0x0\_tt = 0.5339 x0 + -0.1117 x0x0 + -4.5625 x0\_t + -4.9357 x0\_tt + 2.8171 x0x0\_t + -0.3821 x0x0x0\_t + 5.8869 x0x0\_tt | Used PDE library, SINDyPI optimiser  Only velocity (no acceleration plot)  x0\_t is , and x0\_tt is |

21/07: realised adding the noise wasn’t doing what I expected

* If I had x\_train as an array of (1000,1), noise as an array of (1000,), the x\_train\_noisy = x\_train + noise was (1000,1000)
* ???? unsure how to fix as online documentation says it should do what I expect
* Update: worked it out 🡪
  + *np.random.randn(x\_train.shape[0], x\_train.shape[1])*
  + *x\_train = x\_train + 1e-3\*noise*

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| **Test** | **Output** | **Comment** |
| SINDy-PI  Adding noise | (busy long equations) | Recovers noise – model same as true (training data) |
| SINDy  Noisy  Custom library of known terms | Custom Library:  [1, 1, 'x0', 'x1', 'sin(x0)', 'sin(x1)']  Model (w/ noise)  (x0)' = 0.992 x1  (x1)' = 1.646 1 + 1.646 1 + -0.658 x0 + -1.311 x1 |  |
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